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Optimum Production Scheduling for a Beverage Firm Based in Accra

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Abstract: This study, which was conducted in Accra, Ghana, presents a production scheduling problem for a beverage firm based in Accra, all in an attempt to cut down manufacturing cost and increase efficiency. The creation of an optimum production schedule requires the modelling of the scheduling problem as a balanced transportation problem. An important result upon the implementation of the model is the allocation of the optimum level of production necessary to meet a given demand at a minimum cost.

Key words: Linear programming, optimum production, scheduling

INTRODUCTION

Before the beginning of every financial year, many manufacturing companies prepare a production plan. The production plan gives the quantity of goods to be produced for each period during the financial year as well the demand for each period. The production plan can be executed weekly, monthly, quarterly or even yearly depending on the products of the company. Production scheduling is the allocation of available production resources over time to best satisfy some criteria such as quality, delivery time, demand and supply. An optimum production schedule is the production schedule, which efficiently allocates resources over time to best satisfy some set criteria i.e. the plan which allocates the optimum level of production resources necessary to meet a given demand at a minimum cost.

This study shows how we can optimize the production plan of the beverage firm by using the Transportation model.

We present the mathematical formulation of the problem and then solve the problem using the QMS software.

LITERATURE REVIEW

Herrmann (2006) has described the history of production scheduling in manufacturing facilities over the last one hundred (100) years. According to Herrmann (2006), understanding the ways that production scheduling has been done is critical to analyzing existing production scheduling systems and finding ways to improve upon them. The author covered not only the tools used to support decision-making in real-world production scheduling, but also the changes in the production scheduling systems. He extended the work to the first charts developed by Gannt (1973) to advanced scheduling systems that rely on sophisticated algorithms. Through his findings, he was able to help production schedulers, engineers, and researchers understand the true nature of production scheduling in dynamic manufacturing systems and to encourage them to consider how production scheduling systems can be improved even more. The author did not only review the range of concepts and approaches used to improve production scheduling, but also demonstrate their timeless importance.

Lodree and Norman (2006) summarized research related to scheduling personnel where the objective is to optimize system performance while considering human performance limitations and personnel well-being. Topics such as work rest scheduling, job rotation, cross-training, and task learning and forgetting were considered. For these topics, mathematical models and best practices were described.

Pfund and Scott (2006) discussed scheduling and dispatching in one of the most complex manufacturing environments-wafer fabrication facilities. These facilities represent the most costly and time-consuming portion of the semiconductor manufacturing process. After a brief introduction to wafer fabrication operations, the results of a survey of semiconductor manufacturers that focused on the current state of the practice and future needs were presented. They presented a review of some recent dispatching approaches and an overview of recent deterministic scheduling approaches.

MATHEMATICAL FORMULATION

The production problem involves the manufacturing of a single product, which can either be shipped or stored.

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The cost of production and the storage cost of each unit of the products are known. Total cost is made of total production cost plus total storage cost. Storage cost is the cost of carrying one unit of inventory for one time period. The storage cost usually includes insurance cost, taxes on inventory, and a cost due to the possibility of spoilage, theft or obsolescence. The underlying assumptions of the mathematical formulation are:

- Goods produced cannot be allocated prior to being produced.
- Goods produced in a particular month are allocated to the demand in that month or the months ahead.

The production problem is modeled as a balanced transportation problem as follows:

Since production takes place periodically, we consider the time periods in which production takes place as sources $S_1, S_2, \ldots, S_n$ and the time periods in which units will be shipped as destinations $W_1, W_2, \ldots, W_m$. The production capacities a_i at source S_i are taken to be the supplies in a given period i and the demands at the warehouse W_j is d_j.

The problem is to find a production schedule, which will meet all demands at minimum total cost, while satisfying all constraints of production capacity and demands.

Let c_{ij} be the production cost per unit during the time period i plus the storage cost per unit from time period until time j. If we let x_{ij} denote the number of units to be produced during time period i from S_i for allocation during time period j to W_J then for all i and j, $x_{ij} \geq 0$ (since the number of units produced cannot be negative).

i = 1,2 ,...., m and j = 1,2 ,..., n

For each i, the total amount of commodity produced at S_i is:

$$\sum_{j=1}^{n} x_{ij}$$

We shall consider a set of m supply points from which a unit of the product is produced. Since supply point S_i can supply at most a_i units in any given period, we have:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \; i = 1,2, \ldots, m$$

We shall also consider a set of n demand points to which the products are allocated. Since demand points W_J must receive d_j units of the shipped products, we have:

$$\sum_{i=1}^{m} x_{ij} \geq d_j, \; j = 1,2,\ldots,n$$

Since units produced cannot be allocated prior to being produced, C_{ij} is prohibitively large for $i>j$ to force the corresponding x_{ij} to be zero or if allocation is impossible between a given source and destinations, a large cost of M is entered.
The total cost of production then is:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

The objective is to determine the amount of allocated from source to a destination such that the total production costs are minimized.
The model is thus:
Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i = 1,2, \ldots, m \text{ (Supply constraints)}$$

$$\sum_{i=1}^{n} x_{ij} \leq d_i, \; j = 1,2, \ldots, m \text{ (Demand constraints)}$$

$x_{ij} \geq 0$, j = 1,2,...,n i = 1,2,...,m

The non-negativity condition $x_{ij} \geq 0$ is added since negative values for any x_{ij} have no physical meaning.

The production scheduling model was originally formulated by Hitchcock (1941). This was also considered independently by Koopmans (1947).

The Modified Distribution Method (MODI): The formulation above is solved using a method known as the Modified Distribution Method (MODI). An Initial Basic Feasible Solution (IBFS) is required before the application of the MODI. The IBFS can be obtained by the Northwest corner rule, Vogel's approximation method and the least cost method. The IBFS and the MODI will be implemented by the QMS software. MODI aids in obtaining the optimal solution and is established by the following theorem.

Theorem: The theorem states that if we have a basic feasible solution (B.F.S.) consisting of (m+n–1) independent positive allocations and a set of arbitrary numbers u_i and v_j (j = 1,2 ,..., n, i = 1,2 ,..., m) such that $C_{rs} = u_r+v_s$ for all occupied cells (basic variables) then the evaluation corresponding to each empty cell (non-basic variables) (i, j) is given by:

$$\bar{c}_{ij} = c_{ij} - \left(u_i + v_j\right)$$

Proof: Consider the following equations:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, x_{ij} = f \tag{1}$$

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1,2,\ldots,m. \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j, \quad j = 1,2,\ldots.,n. \tag{3}$$

Multiply the ith Eq. of (2) by u_i and the jth Eq. of (3) by v_j (u_i and v_j are arbitrary multipliers and are sometimes called simplex multipliers) to obtain the following:

$$\sum_{j=1}^{n} u_i x_{ij} \le \sum_{i=1}^{m} u_i a_i, \tag{4}$$

and

$$\sum_{i=1}^{m} v_j x_{ij} \ge \sum_{j=1}^{n} v_j d_j \tag{5}$$

Subtract the resultant Eq. (4) and (5) from the objective function Eq. (1) to obtain the modified objective function:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} - \sum_{j=1}^{n} u_i x_{ij} - \sum_{i=1}^{m} v_j x_{ij} = f - \sum_{i=1}^{m} a_i u_i - \sum_{j=1}^{n} d_j v_j \tag{6}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} - u_i - v_j\right)x_{ij} = f - \sum_{i=1}^{m} a_i u_i - \sum_{j=1}^{n} d_j v_j \tag{7}$$

This equation can be written as:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \bar{c}_{ij} x_{ij} = f - f_0 \tag{8}$$

where

$$\bar{c}_{ij} = c_{ij} - u_i - v_j \tag{9}$$

and

$$f_0 = \sum_{i=1}^{m} a_i u_i + \sum_{j=1}^{n} d_j v_j \tag{10}$$

Since the relative cost coefficient $\bar{c}_{ij}$ corresponding to the basic variables (occupied cells) have to be zero, we select u_i and v_j such that:

$$\bar{c}_{ij} = c_{ij} - u_i - v_j = 0 \text{ for basis } x_{ij}$$

Equation (2) and (3) represent (m+n) equations; totally m+n arbitrary multipliers are to be defined. However only (m+n-1) constraint equations are independent and so any one of the Eq. (2) and (3) can be taken as redundant. Since redundant equations really do not exist, their arbitrary multiplier also does not exist.

Hence we have a total of m+n-1 arbitrary multipliers to determine. As the choice of the redundant equation is immaterial, we can set any one of the u_i's or any one of the v_j's to zero.

Once the multipliers u_i and v_j are determined, the relative cost coefficients corresponding to the non-basic variables (unoccupied cells) can be determined easily from Eq. (9) (Amponsah, 2009).

Test for optimality: The following procedure is followed in order to test for optimality

(i) Start with IBFS consisting of (m+n-1) allocations in independent cells.
(ii) Determine a set of (m+n-1) numbers u_i (i = 1,2,…,m) and v_j (j = 1,2,…,n) such that for each occupied cells (r, s) $c_{rs} = u_r + v_s$
(iii) Calculate cell evaluations (unit cost difference) $\bar{c}_{ij}$ for each empty cell (i, j) by using the formula). $\bar{c}_{ij} = c_{ij} - \left(u_i + v_j\right)$
(iv) Examine the matrix of cell evaluation $\bar{c}_{ij}$ for negative entries and conclude that

- If all $\bar{c}_{ij} > 0$ implies Solution is optimal and unique.
- If all $\bar{c}_{ij} \ge 0$ with at least one $\bar{c}_{ij} = 0$ implies Solution is optimal and alternate
- If at least one $\bar{c}_{ij} < 0$ implies Solution is not optimal.

DATA COLLECTION AND ANALYSIS

The capacity data (production plan) for the financial year 2010 of the firm is given in Table 1.

The cost per case of the product is GH¢7.86 and the unit cost of storage is GH¢0.14 per month. With this

Table 1: Capacity data for the beverage firm (in millions)

	Demand	Produce
January	37,287	37,287
February	32,788	32,788
March	39,295	39,295
April	35,313	35,313
May	33,412	33,412
June	31,244	31,244
July	15,888	15,888
August	39,389	39,400
September	39,713	39,800
October	42,237	42,237
November	38,577	38,577
December	47,407	47,507

Table 2: Capacity Data from Table 1 (cases)

	Forecasted demand	Produce
January	4743893130	4743893130
February	4171501272	4171501272
March	4999363868	4999363868
April	4492748092	4492748092
May	4250890585	4250890585
June	3975063613	3975063613
July	2021374046	2021374046
August	5011323155	5012722646
September	5052544529	5063613232
October	5373664122	5373664122
November	4908015267	4908015267
December	6031424936	6044147583
Total	50287913000	55056997000

information we can get the capacity data in cases by dividing each value in Table 1 by 7.86 to get the capacity data in cases, which is given in Table 2.

Scheduling formulation: The formulation takes into account the unit cost of production plus the storage cost C_{ij} (the cost per case is GH¢7.86), the supply a_i at source S_i and the demand d_j at destination for all $i, j \in (1,2,....,12)$

The problem is:
Minimize

$$\sum_{i=1}^{12}\sum_{j=1}^{12} C_{ij}x_{ij}$$

subject to:

$$\sum_{j=1}^{12} x_{ij} \le a_i, i = 1,2 ,...., 12 \text{ (Supply constraints)}$$

$$\sum_{i=1}^{12} x_{ij} \ge d_j, j = 1,2,....,12 \text{ (Demand constraints)}$$

The objective is to determine the amount of x_{ij} allocated from source i to a destination j such that the total production cost $\sum_{i=1}^{12}\sum_{j=1}^{12} C_{ij}x_{ij}$ is minimized.

Thus, we minimize:

$$\sum_{i=1}^{12}\sum_{j=1}^{12} C_{ij}x_{ij}$$

subject to the following supply constraints:

$$x_{11}+x_{12}+x_{13}+x_{14}+x_{15}+x_{16}+x_{17}+x_{18}+x_{19}+x_{1,10}+x_{1,11}+x_{1,12}\le 4743893130$$
$$x_{21}+x_{22}+x_{23}+x_{24}+x_{25}+x_{26}+x_{27}+x_{28}+x_{29}+x_{2,10}+x_{2,11}+x_{2,12}\le 4171501272$$
$$x_{31}+x_{32}+x_{33}+x_{34}+x_{35}+x_{36}+x_{37}+x_{38}+x_{39}+x_{3,10}+x_{3,11}+x_{3,12}\le 4999363868$$
$$x_{41}+x_{42}+x_{43}+x_{44}+x_{45}+x_{46}+x_{47}+x_{48}+x_{49}+x_{4,10}+x_{4,11}+x_{4,12}\le 4492748092$$
$$x_{51}+x_{52}+x_{53}+x_{54}+x_{55}+x_{56}+x_{57}+x_{58}+x_{59}+x_{5,10}+x_{5,11}+x_{5,12}\le 4250890585$$
$$x_{61}+x_{62}+x_{63}+x_{64}+x_{65}+x_{66}+x_{67}+x_{68}+x_{69}+x_{6,10}+x_{6,11}+x_{6,12}\le 3975063613$$
$$x_{71}+x_{72}+x_{73}+x_{74}+x_{75}+x_{76}+x_{77}+x_{78}+x_{79}+x_{7,10}+x_{7,11}+x_{7,12}\le 2021374046$$
$$x_{81}+x_{82}+x_{83}+x_{84}+x_{85}+x_{86}+x_{87}+x_{88}+x_{89}+x_{8,10}+x_{8,11}+x_{8,12}\le 5011323155$$
$$x_{91}+x_{92}+x_{93}+x_{94}+x_{95}+x_{96}+x_{97}+x_{98}+x_{99}+x_{9,10}+x_{9,11}+x_{9,12}\le 5052544529$$

$$x_{10,1}+x_{10,2}+x_{10,3}+x_{10,4}+x_{10,5}+x_{10,6}+x_{10,7}+x_{10,8}+x_{10,9}+x_{10,10}+x_{10,11}+x_{10,12}\le 5373664122$$
$$x_{11,1}+x_{11,2}+x_{11,3}+x_{11,4}+x_{11,5}+x_{11,6}+x_{11,7}+x_{11,8}+x_{11,9}+x_{11,10}+x_{11,11}+x_{11,12}\le 4908015267$$
$$x_{12,1}+x_{12,2}+x_{12,3}+x_{12,4}+x_{12,5}+x_{12,6}+x_{12,7}+x_{12,8}+x_{12,9}+x_{12,10}+x_{12,11}+x_{12,12}\le 6031424936$$

and the following demand constraints

$$x_{11}+x_{21}+x_{31}+x_{41}+x_{51}+x_{61}+x_{71}+x_{81}+x_{91}+x_{10,1}+x_{11,1}+x_{12,1}\le 47438931$$
$$x_{12}+x_{22}+x_{32}+x_{42}+x_{52}+x_{62}+x_{72}+x_{82}+x_{92}+x_{10,2}+x_{11,2}+x_{12,2}\le 47438931$$
$$x_{13}+x_{23}+x_{33}+x_{43}+x_{53}+x_{63}+x_{73}+x_{83}+x_{93}+x_{10,3}+x_{11,3}+x_{12,3}\le 4999363868$$
$$x_{14}+x_{24}+x_{34}+x_{44}+x_{54}+x_{64}+x_{74}+x_{84}+x_{94}+x_{10,4}+x_{11,4}+x_{12,4}\le 4492748092$$
$$x_{15}+x_{25}+x_{35}+x_{45}+x_{55}+x_{65}+x_{75}+x_{85}+x_{95}+x_{10,5}+x_{11,5}+x_{12,5}\le 4250890585$$
$$x_{16}+x_{26}+x_{36}+x_{46}+x_{56}+x_{66}+x_{76}+x_{86}+x_{96}+x_{10,6}+x_{11,6}+x_{12,6}\le 3975063613$$
$$x_{17}+x_{27}+x_{37}+x_{47}+x_{57}+x_{67}+x_{77}+x_{87}+x_{97}+x_{10,7}+x_{11,7}+x_{12,7}\le 2021374046$$
$$x_{18}+x_{28}+x_{38}+x_{48}+x_{58}+x_{68}+x_{78}+x_{88}+x_{98}+x_{10,8}+x_{11,8}+x_{12,8}\le 5012722646$$
$$x_{19}+x_{29}+x_{39}+x_{49}+x_{59}+x_{69}+x_{79}+x_{89}+x_{99}+x_{10,9}+x_{11,9}+x_{12,9}\le 5063613232$$
$$x_{1,10}+x_{2,10}+x_{3,10}+x_{4,10}+x_{5,10}+x_{6,10}+x_{7,10}+x_{8,10}+x_{9,10}+x_{10,10}+x_{11,10}+x_{12,10}\le 5373664122$$
$$x_{1,11}+x_{2,11}+x_{3,11}+x_{4,11}+x_{5,11}+x_{6,11}+x_{7,11}+x_{8,11}+x_{9,11}+x_{10,11}+x_{11,11}+x_{12,11}\le 4908015267$$
$$x_{1,12}+x_{2,12}+x_{3,12}+x_{4,12}+x_{5,12}+x_{6,12}+x_{7,12}+x_{8,12}+x_{9,12}+x_{10,12}+x_{11,12}+x_{12,12}\le 6044147583$$

We shall find the solution to the scheduling formulation by using the QMS software. The QMS implements the MODI to solve the production scheduling formulation.

Using QMS to obtain the BFS and the optimal solution: The QMS is a windows package, which can be used to obtain the optimal solution to a production scheduling problem. Before using the QMS software, we need to create an initial table. This is given in Table 3.

Each cell in Table 3 contains the cost per unit case of the product plus the storage cost. For example, in the first cell i.e., C_{11} the cost is 7.86 whereas in the second cell C_{12} the cost is 8.00 (i.e., 7.86+0.14 = 8.00). A high cost of 10000 is put in cells where production is not feasible. For example in the cell C_{21}, the cost is 10000. This is because one cannot produce in the month of February to meet a demand in January and so a high cost is allocated to that effect

The IBFS and the optimal solution to the problem are given in Table 4 and 5. The IBFS gives the initial allocations of production resources necessary to meet a given demand. Each cell (usually called the occupied cell) contains the respective allocations for each of the periods during the financial year. A cell with no allocation is called an unoccupied cell or an empty cell.

Table 3: Initial table the QMS software uses to generate results

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec	Supply
Jan	7.86	8.00	8.14	8.28	8.42	8.56	8.70	8.84	8.98	9.12	9.26	9.40	4743893130
Feb	10000	7.86	8.00	8.14	8.28	8.42	8.56	8.70	8.84	8.98	9.12	9.26	4171501272
Mar	10000	1000	7.86	8.00	8.14	8.28	8.42	8.56	8.70	8.84	8.98	9.12	4999363868
Apr	10000	10000	10000	7.86	8.00	8.14	8.28	8.42	8.56	8.70	8.84	8.98	4492748092
May	10000	10000	10000	10000	7.86	8.00	8.14	8.28	8.42	8.56	8.70	8.84	4250890585
Jun	10000	10000	10000	10000	10000	7.86	8.00	8.14	8.28	8.42	8.56	8.70	3975063613
July	10000	10000	10000	10000	10000	10000	7.86	8.00	8.14	8.28	8.42	8.56	2021374046
Aug	10000	10000	10000	10000	10000	10000	10000	7.86	8.00	8.14	8.28	8.42	5012722646
Sep	10000	10000	10000	10000	10000	10000	10000	10000	7.86	8.00	8.14	8.28	5063613232
Oct	10000	10000	10000	10000	10000	10000	10000	10000	10000	7.86	8.00	8.14	5373664122
Nov	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	7.86	8.00	4908015267
Dec	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	10000	7.86	6044147583
Dem.	4743893130	4171501272	4999363868	4492748092	4250890585	3975063613	2021374046	5011323155	5052544529	5373664122	4908015267	6031424936	

Table 4: Basic Feasible Solution (BFS) to the scheduling problem generated by the QMS software

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	Dummy
S1	4743893000												
S2		4171501000											
S3			4999364000										
S4				4492748000									
S5					4250890000								
S6						3975064000							
S7							2021374000						
S8								5011323000					1399296
S9									5052545000				11068930
S10										5373664000			
S11											4908015000		
S12												6031425000	12722690

Table 5: Optimal solutions to the production scheduling problem generated by the QMS software

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	Dummy
S1	4743893000	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	0
S2	-9992	4171501000	0	0	0	-1	-1	-1	-1	-1	-1	-1	0
S3	-9992	-9992	4999364000	0	0	0	-1	-1	-1	-1	-1	-1	0
S4	-9992	-9992	-9992	4492748000	0	0	0	-1	-1	-1	-1	-1	0
S5	-9992	-9992	-9992	-9992	4250890000	0	0	0	-1	-1	-1	-1	0
S6	-9992	-9992	-9992	-9992	-9992	3975064000	0	0	0	-1	-1	-1	0
S7	-9992	-9992	-9992	-9992	-9992	-9992	2021374000	0	0	0	-1	-1	0
S8	-9992	-9992	-9992	-9992	-9992	-9992	-9992	5011323000	0	0	0	-1	1399296
S9	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	5052545000	0	0	0	11068930
S10	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	5373664000	0	0	0
S11	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	-9992	4908015000	0	0
S12	-9992	-9992	-9992	-9992	-9992	-9992	-9992		-9992	-9992		6031425000	12722690

The solution in Table 5 is the optimal solution i.e. the Table 5 gives the allocations which minimize the total cost of production. This is so because according to the MODI, if all the factors $\bar{c}_{ij}$ calculated for the empty cells are either negative or zero, then the solution is optimal.

The beverage firm's production plan will have incurred a cost of 432747996420 i.e. cost per unit of production multiplied by the total goods produced for the whole year (7.86(55056997000)).

The optimal solution gave the final total cost of production and is thus:

7.86(474389300 + 4171501000 + 4999364000 + 449274800042508900000+3975064000 + 2021374000 + 5011323000 + 5052545000 + 5373664000 + 4908015000 + 6031425000) = 432747995760

The Optimal solution to the production problem generated by the QMS software is summarized in Table 6.

The S_i with i = 1, 2,...,12 represents the monthly supplies and the D_j with j = 1, 2,…,12 also represents the monthly demands. From the optimum production schedule above, the allocation S_1 to D_1 means use the production in January, 2010 to meet the demand in the same month of January and similarly from S_2 to D_2 also means use the production in the month of February to satisfy the demand in February. The allocations continue till the end of the year. Dummy demands are only created to balance the production problem and so all their allocations do not count.

DISCUSSION

The optimum production schedule presented in Table 5 gives the amount of the product to be allocated to satisfy consumer demand during each period of the financial year. The allocations have been done with the sole objective of minimizing cost.

Table 6: Summary of the optimum production schedule generated by the QMS software

From(supply)	To(demand)	Allocation	Cost per unit	Total Cost
S 1	D 1	4743893000	7.86	293075820000
S 2	D 2	4171501000	7.86	257713680000
S 3	D 3	4999364000	7.86	308858700000
S 4	D 4	4492748000	7.86	277560180000
S 5	D 5	4250890000	7.86	262618320000
S 6	D 6	3975064000	7.86	245577840000
S 7	D 7	2021374000	7.86	124879680000
S 8	D 8	5011323000	7.86	309597540000
S 8	Dummy	1399296	0.00	0
S 9	D 9	5052545000	7.86	312144180000
S 9	Dummy	11068930	0.00	0
S 10	D 10	5373664000	7.86	331982820000
S 11	D 11	4908015000	7.86	303215220000
S 12	D 12	6031425000	7.86	372619020000
S12	Dummy	12722690	0.00	0

The optimal solution gives the allocation that minimizes the total cost of production. On the production schedule, we have an allocation of 4 743 893 000 from January to January i.e. from S1 to D1. That is, in order for the company to make profits or minimize cost, it has to allocate 4 743 893 000 of the goods produced in January to meet the demand in that same month. This allocation will deplete the goods produced in the month of January. Likewise in the month of February 4 171 501 000 of goods produced in that month have to be used to satisfy the demand in that same month. This allocation will deplete the goods produced in the month of February. The schedule continues to give the various allocations until the financial year comes to an end.

For a solution to the production problem to exist, the total demand should be equal to the total supply. The total supply according to Table 2 is 55 056 997 000 and the total demand is 50 287 913 000. Since the total supply is greater than the total demand, a dummy or fictitious demand of 4 769 084 000 (i.e., 55056997000-50287913000) is created to balance the production problem with a cost per unit of zero.

The company has a production of 5 012 722 646 cases of the product in the month of August and 5 063 613 232 in the month of September. However, the optimum schedule revealed that the company should produce only 5 011 323 000 in the month of August and 5 373 664 000 in the month of September. The allocations in the dummy column are not taken into consideration.

CONCLUSION AND RECOMMENDATION

Many production managers or production schedulers go through the process of creating optimum production schedules in an intuitive manner. They obtain these schedules using little or no mathematics which provides a more scientific way of obtaining the optimum schedule. The usage of the scheduling mathematical model to optimize a production schedule is important since production schedulers cannot rely on intuition alone.

The modelling of the production problem as a balanced transportation problem and its specialized methods of solution such as the Northwest corner rule, the least cost method and the Vogel's approximation method developed by Dantzig and Wolfe (1951), which are modifications of the parent simplex algorithm have proven worthwhile in obtaining the optimum schedule. The QMS was used to solve the scheduling formulation.

Ordinarily, the production plan of the firm would have yielded a total production cost of 432747996420, but the optimal production plan or schedule gave a total production cost of 43274799570. This finding is important because the decrease of 660 (i.e., 432747996420-43274799570) in the total cost of production is significant. Furthermore, the optimal solution demonstrated how the reduction will be achieved. (See last paragraph of results description).

The application of the model showed how the monthly allocations should be done in order to reduce the cost of production. It also showed which months the stocks available should be allocated to so that they do not pile up unnecessarily and ultimately reduce the cost of production. The company also is able to produce using regular working time period. This means that overtime or subcontracting is not necessary in reducing the cost of production.

We recommend the usage of the model to determine the optimum level of production to meet a given demand at a minimum cost.

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REFERENCES

Amponsah, S.K., 2009. Lecture notes on operations research. Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi.

Dantzig, G.B. and P. Wolfe, 1951. Decomposition principle for linear programming. J. Oper. Res., 8: 101-111.

Gannt, H.L., 1973. Work, Wages and Profits. Hive Publishing, New York.

Herrmann, J.W., 2006. A History of Production Scheduling. Chap. 1, Handbook of Production Scheduling, Springer, New York.

Hitchcock, F.L., 1941. The distribution of a product from several sources to numerous localities. J. Math. Phys., 20: 224-230.

Koopmans, T.C., 1947. Optimum utilization of the transportation system. Proceedings of the international statistical conference, Washington D.C.

Lodree, E.J. and B.A. Norman, 2006. Scheduling Models for Optimizing Human Performance and Well-Being. Chap. 12, Handbook of Production Scheduling, Springer, New York.

Pfund, M.E. and J.M. Scott, 2006. Semiconductor Manufacturing Scheduling and Dispatching. Chap. 9, Handbook of Production Scheduling, Springer, New York.